


PROBABILITY

CA FDP§

Ref



* Experiment :- A Performance that produces certain results.

* Random Experiment :-

An experiment which produces different results even though it is repeated a large number of times under essentially similar conditions, is called a **random experiment**

Two Outcomes
Head & Tail

Six outcomes
{1, 2, 3, 4, 5, 6}

1. Tossing of a fair coin.



2. Throwing of a balanced die.



3. Drawing of a card from a well-shuffled deck of 52 playing cards.



* Events :- The results or outcome of a random experiment are known as events
It can be combination of outcomes.

i) Simple / Elementary Event

ii) Composite / Compound Event

Simple Event: A simple event in probability is an event with only one possible outcome from a sample space. It's the most basic type of event, where there's no combination of outcomes. For example, flipping a coin and getting heads is a simple event, as it has only one specific outcome.

Single Outcome: A simple event is defined by a single, distinct result.

No Combination: Unlike compound events, a simple event doesn't involve the combination of multiple outcomes.

Probability: The probability of a simple event is the ratio of the number of favorable outcomes (which is 1 for a simple event) to the total number of possible outcomes.

Example: Rolling a die and getting a specific number (e.g., 4) is a simple event.

Compound event: A compound event involves the combination of two or more simple events, leading to multiple possible outcomes.

Example: Rolling a die and getting either an even number or a number greater than 3 is a compound event, as it combines multiple simple events.

* Mutually Exclusive Events or Incompatible Events :-

Mutually exclusive events are events that cannot occur at the same time. If one event happens, the other cannot. They are also known as disjoint events. For example, when flipping a coin, getting heads and getting tails are mutually exclusive events. Simultaneous occurrence is impossible:

→ If event A happens, then event B cannot happen at the same time, and vice versa.

→ Disjoint sets: In probability, mutually exclusive events have no outcomes in common.

→ Probability of both occurring is zero:

→ If A and B are mutually exclusive, the probability of both A and B happening is 0, represented as $P(A \text{ and } B) = P(A \cap B) = 0$.

→ If A and B are mutually exclusive, the probability of either A or B happening is the sum of their individual probabilities:
 $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$.

Examples:

Coin toss: Heads and tails are mutually exclusive.

Rolling a die: Rolling a 1 and rolling a 2 are mutually exclusive.

Choosing a card: Selecting a King and selecting a Queen from a deck of cards are mutually exclusive.

Financial investments: Choosing between two mutually exclusive investment projects.

Traffic: Turning left or right at a fork in the road are mutually exclusive.

* Exhaustive Events :-

Exhaustive events are a set of events where at least one event is guaranteed to occur when an experiment is performed. Essentially, they cover all possible outcomes of an experiment.

Definition: A set of events is exhaustive if, when an experiment is conducted, one of those events must happen.

Sample Space: Exhaustive events encompass the entire sample space of an experiment. The sample space is the set of all possible outcomes.

Example:

When flipping a coin, the outcomes "heads" and "tails" are exhaustive because one of them must occur.

When rolling a six-sided die, the outcomes {1, 2, 3, 4, 5, 6} are exhaustive because one of those numbers will be rolled.

Not necessarily equally likely: While exhaustive events cover all possibilities, the individual events within the set don't necessarily have the same probability of occurring.

* Equally likely Events / Mutually Symmetric / Equi-probable Events :-

Equally likely events are those where each outcome of an event has the same probability of occurring. In simpler terms, there's no bias or preference towards any particular outcome.

Examples

- Tossing a fair coin: When flipping a fair coin, the probability of getting heads is the same as the probability of getting tails (both 1/2 or 50%).
- Rolling a fair six-sided die: Each face of the die (1 through 6) has an equal probability (1/6) of landing face up.
- Drawing a card from a well-shuffled deck: Assuming the deck is fair, each card has an equal chance of being drawn.
- The core feature is that the probability of each possible outcome is the same.

Theoretical Probability: This is based on the structure of the event (e.g., a fair coin has two sides, a die has six sides) and not on observed frequencies.

Absence of Bias: No outcome is favored over another.

Importance: Understanding equally likely events is fundamental in probability theory. It allows for the calculation of probabilities using simple formulas, like the number of favorable outcomes divided by the total number of outcomes.

* Classical definition of Probability :-

The classical definition of probability states that the probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes, assuming all outcomes are equally likely

$$P = \frac{\text{No. of favourable outcomes}}{\text{Total Outcomes}}$$

Limitations :-

- i) Applicable only when the total no. of events are finite
- ii) It can be used only when the events are equally likely or equi-probable

Note :-

i) Probability of an event A lies between 0 and 1, both inclusive

$$0 \leq P(A) \leq 1$$

$P(A) = 0 \rightarrow$ Impossible event ; $P(A) = 1 \rightarrow$ Sure Event

ii) $P(A) + P(A') = 1$

Where $P(A') =$ Not Occurrence of A = $P(A^c) = P(\bar{A})$

iii) Odds in favour of Event A = $\frac{\text{No. of favourable Events}}{\text{No. of unfavourable Events}}$

Odds against Event A = $\frac{\text{No. of unfavourable Events}}{\text{No. of favourable Events}}$

iv) Sample Space : Set of all possible outcomes

Ex-1) A Coin is tossed three times, what is the probability of getting :-

i) 2 heads ii) at least 2 heads (2 or more heads)

Ans $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

i) $P(2 \text{ heads}) = \frac{3}{8}$

ii) $P(\text{at least two heads}) = 1 - [P(0H) + P(1H)]$ or $= P(2H) + P(3H)$
 $= 1 - \left(\frac{1}{8} + \frac{3}{8}\right)$ $= \frac{3}{8} + \frac{1}{8}$
 $= \frac{1}{2}$ $= \frac{1}{2}$

Ex-2) A dice is rolled twice, what is the probability of getting a difference of 2 points ?

Ans favourable cases : (1,3), (2,4), (3,1), (3,5), (4,2), (4,6), (5,3), (6,4)

$$P = \frac{8}{36} = \frac{2}{9} \text{ Ans}$$

Ex-3) Two dice are thrown simultaneously. find the probability that the sum of points on the two dice would be 7 or more.

Ans Sum 7 : (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

Sum 8 : (2,6), (3,5), (4,4), (5,3), (6,2)

Sum 9 : (3,6), (4,5), (5,4), (6,3)

Sum 10 : (4,6), (5,5), (6,4)

Sum 11 : (5,6), (6,5)

Sum 12 : (6,6)

$$\text{Required probability} = \frac{21}{36} = \frac{7}{12}$$

Ex-4) What is the chance of picking a Spade or an ace not of Spade from a pack of 52 Cards ?

Ans A: Picking a Spade $P(A) = \frac{13}{52}$

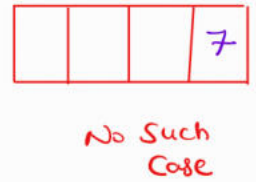
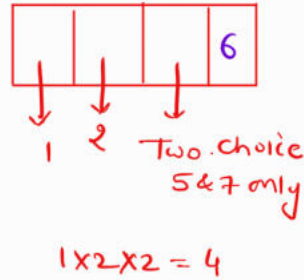
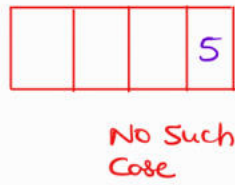
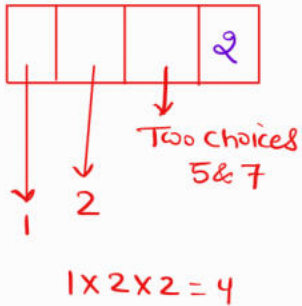
B: Ace not of Spade $P(B) = \frac{3}{52}$

$$A \cap B = \phi \quad P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{16}{52} = \frac{4}{13} \text{ Ans}$$

Ex-5) find the probability that a four digit no. comprising the digits 2, 5, 6 and 7 would be divisible by 4.

Ans A No. to be divisible by 4 \rightarrow last two digits divisible by 4
or
last two digits are zero.



$$\text{Req. prob.} = \frac{4+4}{4!} = \frac{8}{24} = \frac{1}{3}$$

Ex-6) A Committee of 7 members is to be formed from a group comprising 8 males & 5 women. What is the probability that the Committee would comprise

- i) 2 women
- ii) at least 2 women

Ans Total Cases = $^{13}C_7$

$$i) P(2 \text{ women}) = \frac{{}^5C_2 \cdot {}^8C_5}{{}^{13}C_7} = \frac{\cancel{5} \cdot \cancel{4} \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6}}{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5} \cdot \cancel{6}} = \frac{140}{429} \text{ Ans}$$

$$ii) P(W \geq 2) = 1 - [P(W=0) + P(W=1)]$$

$$= 1 - \left(\frac{{}^8C_7}{{}^{13}C_7} + \frac{{}^8C_6 \cdot {}^5C_1}{{}^{13}C_7} \right)$$

$$= 1 - \left(\frac{148}{\cancel{13} \cdot \cancel{12} \cdot \cancel{11} \cdot \cancel{10} \cdot \cancel{9} \cdot \cancel{8}} \cdot \frac{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5} \cdot \cancel{6}}{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5} \cdot \cancel{6}} \right) = 1 - \frac{37}{429} = \frac{392}{429} \text{ Ans}$$

* Relative frequency definition of probability :-

Let an event A occurs f_A times, then the limiting value of the ratio of f_A to n as n tends to infinity is defined as

$$P(A) = \lim_{n \rightarrow \infty} \frac{f_A}{n}$$

Definition

Frequency in mathematics is a measure of how often a quantity is present and represents the chances of occurrence of that quantity.

Formula

$$\text{Relative Frequency} = \frac{\text{Subgroup Count}}{\text{Total Count}} = \frac{f}{n}$$

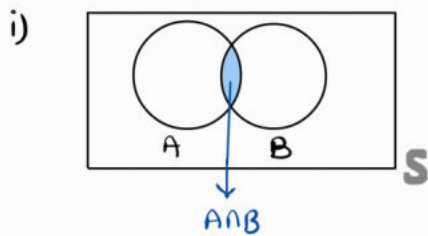
Example :- The following data relate to the distribution of wages of a group of workers:

Wages in ₹:	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers:	15	23	36	42	17	12	5

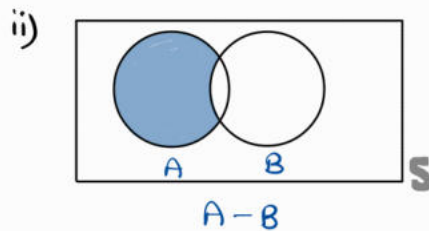
If a worker is selected at random from the entire group of workers, what is the probability that

- (a) his wage would be less than ₹ 50? $\frac{0}{150} = 0$
 (b) his wage would be less than ₹ 80? $\frac{74}{150}$
 (c) his wage would be more than ₹ 100? $\frac{17}{150}$
 (d) his wages would be between ₹ 70 and ₹ 100? $\frac{95}{150}$

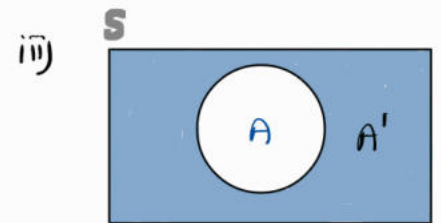
* Set theory formula's :-



$$A \cap B = \{x \in A \text{ and } x \in B\}$$



$$A - B = \{x \in A \text{ and } x \notin B\}$$



$$A' = \{x \notin A \text{ and } x \in S\}$$

Note :-

i) Two events A and B are mutually exclusive if $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$

ii) Three events A, B and C are mutually exclusive if $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

iii) Two events A and B are exhaustive if $P(A \cup B) = 1$

iv) Three events A, B and C are exhaustive if $P(A \cup B \cup C) = 1$

v) Three events A, B and C are equally likely if $P(A) = P(B) = P(C)$

vi) $A \subseteq S$ then $P(A) \geq 0$ and $P(S) = 1$; $S \rightarrow$ Sample Space

vii) for any sequence of mutually exclusive events A_1, A_2, A_3, \dots we have

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) + \dots$$

Ex :- Three events A, B, C are mutually exclusive, exhaustive and equally likely find the prob. of complementary event of A.

Ans $P(A \cup B \cup C) = P(A) + P(B) + P(C)$ [Mutually Exclusive]

$P(A \cup B \cup C) = 1$ [Mutually Exhaustive]

$P(A) = P(B) = P(C) = k$ (say) [Equally likely]

So, $1 = k + k + k \Rightarrow k = 1/3$

$P(A) = 1/3$ So, $P(A') = 1 - P(A) = 1 - 1/3 = 2/3$ Ans

De Morgan's law
 $P(A' \cap B') = P(A \cup B)'$
 &
 $P(A' \cup B') = P(A \cap B)'$

Note :-

i) for any two mutually Exclusive events A and B, probability that either A or B occurs is given by the sum of individual probabilities of A and B.

$$P(A \cup B) = P(A+B) = P(A) + P(B) \quad \text{or} \quad P(A \cap B) = 0 \quad ; \quad \text{where A \& B are mutually Exclusive}$$

ii) for any $k (> 2)$ mutually exclusive events A_1, A_2, \dots, A_k the probability that at least one of them occurs is given by the sum of individual probabilities of the k events

$$\text{i.e. } P(A_1 \cup A_2 \cup A_3 \dots \cup A_k) = P(A_1) + P(A_2) + \dots + P(A_k)$$

iii) for any two events A and B, the probability that either A or B occurs is given by the sum of individual probabilities of A and B less the probability of simultaneous occurrence of the events A and B.

$$\text{i.e. } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

iv) for any three events A, B and C, the probability that at least one of the events occurs is given by

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\text{v) } (A-B) \cup (B-A) = (A \cup B) - (A \cap B)$$

$$\text{vi) } (A-B) \cap (B-A) = \phi$$

$$\text{vii) } P(A-B) = P(A) - P(A \cap B) = P(A \cap B')$$

Ex → A no. is selected from the first 25 natural numbers, what is the probability that it would be divisible by 4 or 7?

$$\text{A: divisible by 4} = \{4, 8, 12, \dots, 24\}$$

$$\text{B: divisible by 7} = \{7, 14, 21\}$$

$$A \cap B = \phi$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{25} + \frac{3}{25} - \frac{0}{25} = \frac{9}{25}$$

Ex → A no. is selected at random from the first 1000 natural numbers. What is the prob. that it would be multiple of 5 or 9?

$$\text{A: multiple of 5} = \{5, 10, 15, \dots, 1000\} \quad \text{Total} = 200$$

$$\text{B: multiple of 9} = \{9, 18, 27, \dots, 999\} \quad \text{Total} = 111$$

$$\text{A \cap B: multiple of 5 and 9} = \{45, 90, 135\} \quad \text{Total} = 22$$

$$P(A \cup B) = \frac{200}{1000} + \frac{111}{1000} - \frac{22}{1000}$$

$$= \frac{289}{1000} = 0.289 \quad \text{A.}$$

Ex → The prob. that an accountant's job applicant has a B.Com degree is 0.85, that he is C.A. is 0.30 and that he is both B.Com and C.A. is 0.25 out of 500 applicants. How many would be B.Com or C.A.?

Ans: A: B.Com degree $P(A) = 0.85$
 B: C.A. $P(B) = 0.30$

$$P(A \cap B) = 0.25$$

$$P(A \cup B) = 0.85 + 0.30 - 0.25 = 0.90 \quad \& \quad n(A \cup B) = (0.90)(500) = 450 \quad \underline{Ans}$$

Q $P(A-B) = \frac{1}{5}$; $P(A) = \frac{1}{3}$ & $P(B) = \frac{1}{2}$

what is the probability that out of the two events A and B, only B would occur

Ans $P(A-B) = \frac{1}{5}$

$$P(A) - P(A \cap B) = \frac{1}{5} \Rightarrow P(A \cap B) = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$P(B \text{ only}) = P(B-A) = P(B) - P(A \cap B) \\ = \frac{1}{2} - \frac{2}{15} = \frac{11}{30} \quad \underline{Ans}$$

Example :- There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

Ans $P(A) = 0.80$ $P(A \cap B) = 0.46$ $P(A \cap B \cap C) = 0.26$
 $P(B) = 0.60$ $P(B \cap C) = 0.32$
 $P(C) = 0.50$ $P(C \cap A) = 0.48$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \\ = 0.8 + 0.6 + 0.5 - 0.46 - 0.32 - 0.48 + 0.26 \\ = 0.9 \quad \underline{Ans}$$

* Compound theorem of Probability :-

The compound theorem of probability, also known as the multiplicative rule of probability, deals with finding the probability of two or more independent events occurring together. It states that the probability of both events A and B occurring is the product of their individual probabilities:

$$P(A \text{ and } B) = P(A) \cdot P(B) \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

Independent Events: Independent events are those where the outcome of one event does not influence the outcome of the other. For example, flipping a coin twice is an example of independent events, as the result of the first flip does not affect the second flip.

Two events A and B are said to be independent events if

$$P(A \cap B) = P(A) \cdot P(B)$$

In a similar manner, $P(A \cap B \cap C) = P(A)P(B)P(C)$ for A, B & C independent events.

the probability of simultaneous occurrence of k events A_1, A_2, \dots, A_k

We have $P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_k) = P(A_1)P(A_2) \dots P(A_k)$

Basal Note :-

If A and B are independent events i.e. $P(A \cap B) = P(A)P(B)$ then

- i) A' and B' are also independent events i.e. $P(A' \cap B') = P(A')P(B')$
- ii) A' and B are also independent events i.e. $P(A' \cap B) = P(A')P(B)$
- iii) A and B' are also independent events i.e. $P(A \cap B') = P(A)P(B')$

Conditional Probability :-

conditional probability is a measure of the probability of an event occurring, given that another event is already known to have occurred. This particular method relies on event A occurring with some sort of relationship with another event B.

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$



Prob. of the event B given that the event A has already occurred

or

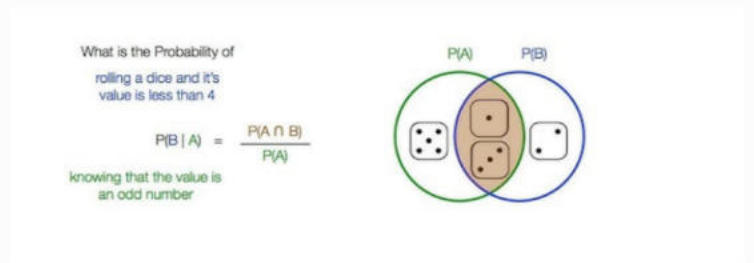
Conditional prob. of B given A

here $P(A) > 0$ i.e. A is not an impossible event

Similarly, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$P(A|B) = P(A)$
implies A and B are independent Events



Star Note :-

$$P(A' \cap B) = P(B) - P(A \cap B)$$

So,

$$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

for any three events A, B and C the probability that they occur jointly is given by

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B) ; \text{ provided } P(A \cap B) > 0$$

Q. Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?

A. $P(\text{Target hit}) = 1 - P(\text{Not hit})$
 $= 1 - P(\bar{R})P(\bar{D})$
 $= 1 - \frac{4}{9} \cdot \frac{5}{11}$
 $= 1 - \frac{20}{99} = \frac{79}{99}$

$P(\text{Target Hit}) = P(R)P(\bar{D}) + P(\bar{R})P(D)$
 $+ P(R)P(D)$
 $= \frac{5}{9} \cdot \frac{5}{11} + \frac{4}{9} \cdot \frac{6}{11} + \frac{5}{9} \cdot \frac{6}{11}$
 $= \frac{79}{99}$ A.

Q $P(A) = \frac{2}{3}$; $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{5}{6}$

find $P(A|B)$, $P(B|A)$, $P(A'|B)$, $P(A|B')$, $P(A'|B')$

A $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
 $= \frac{2}{3} + \frac{3}{5} - \frac{5}{6} = \frac{20 + 18 - 25}{30} = \frac{13}{30}$

i) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{13}{30}}{\frac{3}{5}} = \frac{13}{18}$ ii) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{13}{30}}{\frac{2}{3}} = \frac{13}{20}$

iii) $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{3}{5} - \frac{13}{30}}{\frac{3}{5}} = \frac{\frac{6 - 13}{30}}{\frac{3}{5}} = \frac{5}{18}$

iv) $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{\frac{2}{3} - \frac{13}{30}}{1 - \frac{3}{5}} = \frac{\frac{4 - 13}{30}}{\frac{2}{5}} = \frac{7}{12}$

v) $P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - \frac{5}{6}}{1 - \frac{3}{5}} = \frac{\frac{1}{6}}{\frac{2}{5}} = \frac{5}{12}$
 (de-Morgan's)

Q The odds in favour of an event is 2:3 and the odds against another event is 3:7. find the Prob. that only one of the two events occurs.

A $P(A) = \frac{2}{5}$, $P(B) = \frac{7}{10}$

$P(\text{only one event occurs}) = P(A)P(\bar{B}) + P(B)P(\bar{A})$
 $= \frac{2}{5} \cdot \frac{3}{10} + \frac{7}{10} \cdot \frac{3}{5} = \frac{6}{50} + \frac{21}{50} = \frac{27}{50}$ A

Q There are 3 boxes with the following composition

Box I : 7R 5W 4B Balls

Box II : 5R 6W 3B Balls

Box III : 4R 3W 2B Balls

R: Red
 W: white
 B: Blue

One of the boxes is selected at random & a ball is drawn. what is the prob. that the drawn ball is Red.

A $P(R) = P(\text{Box I}) \cdot P(R|\text{Box I}) + P(\text{Box II}) \cdot P(R|\text{Box II}) + P(\text{Box III}) \cdot P(R|\text{Box III})$
 $= \frac{1}{3} \cdot \frac{7}{16} + \frac{1}{3} \cdot \frac{5}{14} + \frac{1}{3} \cdot \frac{4}{9} = \frac{1249}{3024}$

Q In a group of 20M and 15F, 12M and 8F are Service holders. what is the Prob. that a person selected at random from the group is a service holder given that the selected person is a male?

A $P\left(\begin{matrix} \text{Service} \\ \text{holder} \\ A \end{matrix} \middle| \begin{matrix} \text{Person} \\ \text{male} \\ B \end{matrix}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{12}{35}}{\frac{20}{35}} = \frac{12}{20} = 0.6$

$A \cap B$: Male + Service holder = 12

B : Male = 20

Q A pair of dice is thrown together and the sum of points of two dice is noted to be 10. What is the prob. that one of the two dice has shown point 4.

$$A) P(\text{one dice shows } 4 \mid \text{Sum is } 10) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{3}{36}} = \frac{2}{3}$$

\downarrow \downarrow
 A B

$$A \cap B : \text{Sum } 10 + \text{dice shows } 4 : \{ (4,6), (6,4) \}$$

$$B : \text{Sum is } 10 : \{ (4,6), (5,5), (6,4) \}$$

Q If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?

A Every ball have 3 choices.

after bag I contain 3 balls, Remaining balls have two choices only

$$\text{So, Req. Prob} = \frac{{}^8C_3 \cdot 2^5}{3^8}$$

Q There are two urns containing 5 red and 6 white balls and 3 red and 7 white balls respectively. If two balls are drawn from the first urn without replacement and transferred to the second urn and then a draw of another two balls is made from it, what is the probability that both the balls drawn are red?

A A: Both balls transferred are red (2nd Urn have 5R 7W) $P(A) = \frac{{}^5C_2}{{}^{11}C_2}$
 B: _____ white (2nd Urn have 3R 9W) $P(B) = \frac{{}^6C_2}{{}^{11}C_2}$
 C: _____ of different color (2nd Urn have 4R 8W) $P(C) = \frac{{}^5C_1 \cdot {}^6C_1}{{}^{11}C_2}$
 D: Draw another two balls are red

$$P(D) = P(D|A) + P(D|B) + P(D|C)$$

$$= \frac{{}^5C_2}{{}^{11}C_2} \cdot \frac{{}^5C_2}{{}^{12}C_2} + \frac{{}^6C_2}{{}^{11}C_2} \cdot \frac{{}^3C_2}{{}^{12}C_2} + \frac{{}^5C_1 \cdot {}^6C_1}{{}^{11}C_2} \cdot \frac{{}^4C_2}{{}^{12}C_2} = \frac{65}{726}$$

Q A lot of 10 electronic components is known to include 3 defective parts. If a sample of 4 components is selected at random from the lot, what is the probability that this sample does not contain more than one defectives?

$$A) P(\text{Not more than one defective}) = P(\text{Zero def.}) + P(\text{one def.})$$

$$= \frac{{}^7C_4}{{}^{10}C_4} + \frac{{}^3C_1 \cdot {}^7C_3}{{}^{10}C_4}$$

$$= \frac{35}{210} + \frac{3 \cdot 35}{210} = \frac{2}{3}$$

3 Def
7 Non-Def
Total = 10 ✓

Q The independent probabilities that the three sections of a costing department will encounter a computer error are 0.2, 0.3 and 0.1 per week respectively. What is the probability that there would be

A, B, C (let)

(i) at least one computer error per week? $= 1 - P(\text{No Error}) = 1 - \frac{8}{10} \cdot \frac{7}{10} \cdot \frac{9}{10} = 1 - \frac{504}{1000} = 0.496$

(ii) one and only one computer error per week?

$$= P(A)P(\bar{B})P(\bar{C}) + P(B)P(\bar{A})P(\bar{C}) + P(C)P(\bar{A})P(\bar{B})$$

$$= \frac{2}{10} \cdot \frac{7}{10} \cdot \frac{9}{10} + \frac{3}{10} \cdot \frac{8}{10} \cdot \frac{9}{10} + \frac{1}{10} \cdot \frac{8}{10} \cdot \frac{7}{10}$$

$$= \frac{126 + 216 + 56}{1000} = \frac{398}{1000} = 0.398$$

Q

Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates. What is the probability that Mr. Roy would be selected?

A $P(\text{Roy Selects}) = 1 - P(\text{Roy Not select})$

$$= 1 - \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{9}{10} = 1 - \frac{72}{150} = \frac{78}{150} = \frac{13}{25}$$

Q

There are three boxes with the following compositions :

Box \ Colour	Blue	Red	White	Total
I	5	8	10	23
II	4	9	8	21
III	3	6	7	16

One ball is drawn from each box. What is the probability that they would be of the same colour?

A $P(\text{Same Color}) = P(BBB) + P(RRR) + P(WWW)$

$$= \frac{5}{23} \cdot \frac{4}{21} \cdot \frac{3}{16} + \frac{8}{23} \cdot \frac{9}{21} \cdot \frac{6}{16} + \frac{10}{23} \cdot \frac{8}{21} \cdot \frac{7}{16}$$

$$= \frac{60 + 432 + 560}{23 \cdot 21 \cdot 16} = \frac{1052}{7728} = \frac{263}{1932}$$

A

* Random Variable :-

A random variable is a variable whose numerical value is determined by the outcome of a random phenomenon. A probability distribution describes the likelihood of all possible values that a random variable can take.

A random variable assigns a numerical value to each possible outcome of a random experiment.

Types: Discrete: Takes on countable values (e.g., the number of heads when flipping a coin 10 times).

Continuous: Can take any value within a given range (e.g., the height of a student).

Examples: The outcome of a die roll (discrete).

The temperature of a room (continuous). , height , weight

* Probability Distribution :-

A probability distribution describes the likelihood of each possible value of a random variable.

Discrete Probability Distributions:

Probability Mass Function (PMF): Assigns probabilities to each distinct value of a discrete random variable.

Example: For a fair six-sided die, the PMF would assign a probability of 1/6 to each face (1, 2, 3, 4, 5, 6).

Continuous Probability Distributions:

Probability Density Function (PDF): Describes the relative likelihood of a continuous random variable falling within a specific range of values.

Example: The normal distribution, often represented by a bell curve, is a common PDF.

Key Properties:

1. Probabilities are always non-negative. $P_i \geq 0 \forall i$
2. The sum of all probabilities in a discrete distribution equals 1 (or the integral of the PDF over all possible values equals 1 for continuous distributions). $\sum P_i = 1$ (over all i)
3. Probability distributions can be used to calculate probabilities of events and to summarize the behavior of random variables.

Probability distribution of random variable x is given by

X	x_1	x_2	x_3	x_4	x_5	x_6	...	x_n	Total
P	P_1	P_2	P_3	P_4	P_5	P_6	...	P_n	1

Ex → Prob. distⁿ of Head when a coin is tossed twice

X : No. of Heads ; $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

X can take values 0, 1, 2, 3

So, prob. distⁿ can be written as →

X	0	1	2	3	Total
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

Note :-

i) If X is a discrete variable and if such a function $f(x)$ really exists, then $f(x)$ is known as PMF of X , satisfying the conditions

$$f(x) \geq 0$$

$$\sum_x f(x) = 1 \quad \text{where } f(x) = P(X=x)$$

ii) If X is a continuous random variable defined over an interval $[\alpha, \beta]$; $\beta > \alpha$ then X can assume infinite no. of values from its interval and instead of assigning individual prob. to every mass point x , we assign probabilities to interval of values, then $f(x)$ is known as PDF of X , satisfying the conditions

$$f(x) \geq 0 \quad \forall x \in [\alpha, \beta]$$

$$\int_{\alpha}^{\beta} f(x) dx = 1$$

and the probability that x lies b/w two specified values a and b ,

where $\alpha \leq a < b \leq \beta$ is given by $\int_a^b f(x) dx$

* Expected Value of a Random Variable :-

Expected value or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.

If a random variable x assumes n values x_1, x_2, \dots, x_n with corresponding probabilities p_1, p_2, \dots, p_n

then the expected value of x is given by

$$E(x) = \mu = \sum_{i=1}^n p_i x_i$$

$$E(x^2) = \sum_{i=1}^n p_i^2 x_i^2$$

$$E(g(x)) = \sum_{i=1}^n p_i g(x_i) \quad ; \quad g(x) \rightarrow \text{monotonic function}$$

Variance \rightarrow

$$\begin{aligned} \text{Var}(x) = \sigma^2 &= E(x - \mu)^2 \\ &= E(x^2) - (E(x))^2 \\ &= E(x^2) - \mu^2 \end{aligned}$$

Proof:-

$$\begin{aligned} \mu &= E(x) \\ E(x - \mu)^2 &= E(x^2 + \mu^2 - 2\mu x) \\ &= E(x^2) + E(\mu^2) - 2 \cdot E(\mu x) \\ \text{here } \mu &\rightarrow \text{Mean} \rightarrow \text{Constant} \\ &= E(x^2) + \mu^2 - 2 \cdot \mu \cdot E(x) \\ &= E(x^2) + \mu^2 - 2\mu \cdot \mu \\ &= E(x^2) - (E(x))^2 \end{aligned}$$

Standard deviation :-

$$\text{S.D}(x) = \sigma = \sqrt{E(x^2) - (E(x))^2}$$

If $y = a + bx$ then

$$\mu_y = a + \mu_x \quad \text{and} \quad \overset{\text{S.D.}}{\sigma_y} = |b| \cdot \sigma_x \quad \text{and} \quad \overset{\text{Variance}}{\sigma_y^2} = |b|^2 \sigma_x^2$$

again,

when x is a discrete random variable with probability mass function $f(x)$, then its expected value is given by

$$E(x) = \mu = \sum_x x f(x)$$

$$E(x^2) = \sum_x x^2 f(x)$$

$$\sigma^2 = E(x^2) - (E(x))^2 = E(x^2) - \mu^2$$

for a continuous random variable x defined in $(-\infty, \infty)$, its expected value (Mean)

and variance are given by

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx \quad \text{and} \quad \text{Var}(x) = \sigma^2 = E(x^2) - E(x)^2 = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

Properties of Expected Values :-

- (i) Expectation of a constant k is k
i.e. $E(k) = k$ for any constant k .
- (ii) Expectation of sum of two random variables is the sum of their expectations.
i.e. $E(x + y) = E(x) + E(y)$ for any two random variables x and y .
- (iii) Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
i.e. $E(kx) = k.E(x)$ for any constant k
- (iv) Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.
i.e. $E(xy) = E(x) \times E(y)$
Whenever x and y are independent.

Q. An unbiased coin is tossed three times. find the Expected Value of the no. of heads and also its Standard deviation.

Ans. $E(X) = \sum P_i x_i$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$= \frac{3}{2} = 1.5 \text{ (Mean)}$$

$$E(X^2) = \sum P_i x_i^2$$

$$= 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = 3$$

$$\text{Var}(X) = \sigma^2 = E(X^2) - (E(X))^2$$

$$= 3 - (1.5)^2 = 3 - 2.25 = 0.75$$

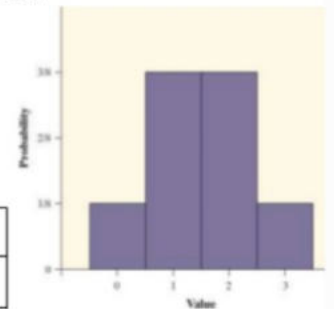
$$\text{S.D} = \sigma = \sqrt{0.75} = 0.87 \text{ (S.D.)}$$

Consider tossing a fair coin 3 times.

Define X = the number of heads obtained

$X = 0$: TTT
 $X = 1$: HTT THT TTH
 $X = 2$: HHT HTH THH
 $X = 3$: HHH

Value	0	1	2	3
Probability	1/8	3/8	3/8	1/8



Q.

A random variable has the following probability distribution:

X:	4	5	7	8	10
P:	0.15	0.20	0.40	0.15	0.10

Find $E[x - E(x)]^2$. Also obtain $v(3x - 4)$

Ans. $E(X - E(X))^2 = E(X - \mu)^2 = \sigma^2$

$$= E(X^2) - E(X)^2$$

$$= \sum x_i^2 P_i - (\sum x_i P_i)^2$$

$$= (16)(0.15) + (25)(0.20) + (49)(0.40) + (64)(0.15) + (100)(0.10) - (6.6)^2$$

$$= 2.4 + 5 + 19.6 + 9.6 + 10 - 43.56$$

$$= 3.04 = \text{Var}(X)$$

$\sum x_i P_i = \mu = 0.6 + 1 + 2.8 + 1.2 + 1$
 $= \mu = 6.6$

$$\begin{aligned} \text{and } \text{Var}(3x-4) &= 3^2 \cdot \text{Var}(x) \\ &= 9 \cdot (3.04) \\ &= 27.36 \quad \underline{\underline{A_1}} \end{aligned}$$

Q

A random variable x has the following probability distribution :

X	:	0	1	2	3	4	5	6	7
P(X)	:	0	2k	3k	k	2k	k ²	7k ²	2k ² +k

Find (i) the value of k

(ii) $P(x < 3)$

(iii) $P(x \geq 4)$

(iv) $P(2 < x \leq 5)$

A₁ Sum of all probabilities = 1
 $\Rightarrow 9k + 10k^2 = 1 \Rightarrow 10k^2 + 9k - 1 = 0$
 $k = -1, k = \frac{1}{10}$
 \times \checkmark

$$\begin{aligned} P(x < 3) &= P(x=0) + P(x=1) + P(x=2) \\ &= 5k = \frac{5}{10} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} P(x \geq 4) &= 1 - (P(x=0) + P(x=1) + P(x=2) + P(x=3)) \\ &= 1 - 6k = 1 - \frac{6}{10} = \frac{4}{10} = \frac{2}{5} \end{aligned}$$

$$\begin{aligned} P(2 < x \leq 5) &= P(x=3) + P(x=4) + P(x=5) \\ &= 3k + k^2 \\ &= \frac{3}{10} + \frac{1}{100} = \frac{31}{100} \quad \underline{\underline{A_1}} \end{aligned}$$

Q

A dice is thrown repeatedly till a 'six' appears. Write down the sample space. Also find the expected number of throws.

A₁ $X = \text{No. of throws necessary to get a six}$

let $P = P(\text{six appears}) = \frac{1}{6}$

$q = P(\text{six not appears}) = \frac{5}{6}$

Prob. distribution is

x	1	2	3	4	and so on.
Prob.	p	q^2p	q^2p	q^3p		

$$\begin{aligned} E(x) &= \sum x_i p_i \\ &= p + 2q^2p + 3q^2p + 4q^3p + \dots \\ &= p(1 + 2q + 3q^2 + 4q^3 + \dots) \\ &= p \cdot \frac{1}{(1-q)^2} \\ &= \frac{\frac{1}{6}}{\left(\frac{1-5}{6}\right)^2} = \frac{\frac{1}{6}}{\frac{1}{36}} = 6 \quad \underline{\underline{A_1}} \end{aligned}$$

TRICK :-

$$\begin{aligned} S &= 1 + 2x + 3x^2 + 4x^3 + \dots \\ xS &= x + 2x^2 + 3x^3 + \dots \\ (1-x)S &= 1 + x + x^2 + \dots \\ (1-x)S &= \frac{1}{1-x} \\ S &= \frac{1}{(1-x)^2} \end{aligned}$$

Q

A number is selected at random from a set containing the first 100 natural numbers and another number is selected at random from another set containing the first 200 natural numbers. What is the expected value of the product?

Ans: X: No. Selected from the first 100 natural no.

X can take values 1, 2, 3, ..., 100

with same probability $\frac{1}{100}$

Y: No. Selected from the first 200 Natural no.'s

Y can take values 1, 2, ..., 200

with same probability $\frac{1}{200}$

Prob. distⁿ is →

X	1	2	3	...	100
P	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$...	$\frac{1}{100}$

Prob. distⁿ is →

Y	1	2	3	...	200
P	$\frac{1}{200}$	$\frac{1}{200}$	$\frac{1}{200}$...	$\frac{1}{200}$

$$E(X) = \sum x_i p_i$$

$$E(X) = \frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \dots + \frac{100}{100}$$

$$= \frac{(100)(101)}{2 \cdot 100} = \frac{101}{2}$$

$$E(Y) = \sum y_i p_i$$

$$= \frac{1}{200} + \frac{2}{200} + \frac{3}{200} + \dots + \frac{200}{200}$$

$$= \frac{(200)(201)}{2(200)} = \frac{201}{2}$$

$$\text{So, } E(XY) = E(X)E(Y)$$

$$= \frac{101}{2} \cdot \frac{201}{2} = \frac{20301}{4} = 5075.25 \text{ Ans.}$$

Q

Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation? If he is asked to pay ₹ 400 for participating in the game, would he consider it a fair game and participate?

Ans.

3W 5R
↓
2 Balls drawn

$$WW \rightarrow P(WW) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28} \quad (\text{Both white then he get 1000 rs.})$$

$$WR \rightarrow P(WR) = \frac{{}^3C_1 \cdot {}^5C_1}{{}^8C_2} = \frac{15}{28} \quad (\text{one white one Red then he get 700 rs.})$$

$$RR \rightarrow P(RR) = \frac{{}^5C_2}{{}^8C_2} = \frac{10}{28} \quad (\text{both red then he get 400 rs.})$$

X → Amount then Prob. distⁿ is →

X	1000 ₹	700 ₹	400 ₹
P	$\frac{3}{28}$	$\frac{15}{28}$	$\frac{10}{28}$

$$\text{Expected amount} = \frac{3000}{28} + \frac{10500}{28} + \frac{4000}{28} = \frac{17500}{28} = 625 > 400$$

fair ✓ Participate ✓

Q

A box contains 12 electric lamps of which 5 are defective. A man selects three lamps at random. What is the expected number of defective lamps in his selection?

Ans: X: No. of defective lamps

X: 0, 1, 2, 3



Prob distⁿ →

X	0	1	2	3
P(X)	$\frac{{}^5C_0 \cdot 7C_3}{12C_3} = \frac{35}{220}$	$\frac{{}^5C_1 \cdot 7C_2}{12C_3} = \frac{105}{220}$	$\frac{{}^5C_2 \cdot 7C_1}{12C_3} = \frac{70}{220}$	$\frac{{}^5C_3}{12C_3} = \frac{10}{220}$

$$E(X) = \sum x_i p_i$$

$$= 0 + \frac{105}{220} + \frac{140}{220} + \frac{30}{220} = \frac{275}{220} = 1.25 \quad \underline{A_1}$$

11

In a business venture, a man can make a profit of ₹ 50,000 or incur a loss of ₹ 20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?

A₁

X	50000	-20000
P	0.75	0.25

$$\begin{aligned} E(X) &= (50000)(0.75) + (-20000)(0.25) \\ &= 37500 - 5000 \\ &= 32500 \text{ ₹} \quad \underline{A_1} \end{aligned}$$